Year 12 Maths Methods Test 1, 2017

Differentiation Techniques and Applications of Differentiation

Name:_____

Section 1: Resource Free 30 marks 30 minutes

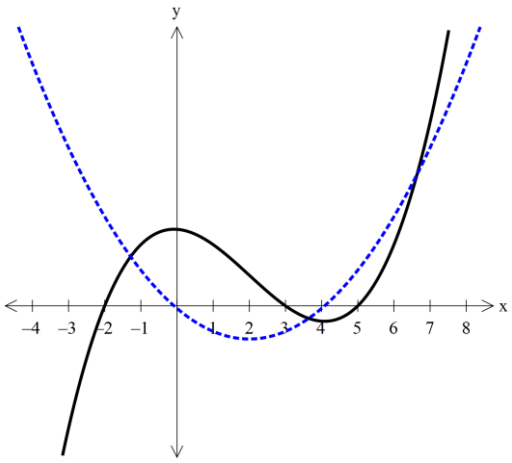
QUESTION 1 [3, 1, 2, 7 marks]

a) If $f(x)=\frac{1}{2x^2}$, evaluate $f''(-1)$

b) Find g'(x), if $g(x)=(1+2x-2x^3)(x^2-1)$; do not simplify your answer

c) Use the chain rule to differentiate $\frac{2}{(x^3+2)^4}$; apply basic simplification

d) The 1st and 2nd derivative function of a function is shown. The x-coordinates of points where various features of the original function occur are shown below. State the nature of each of these points:



i. x = -2

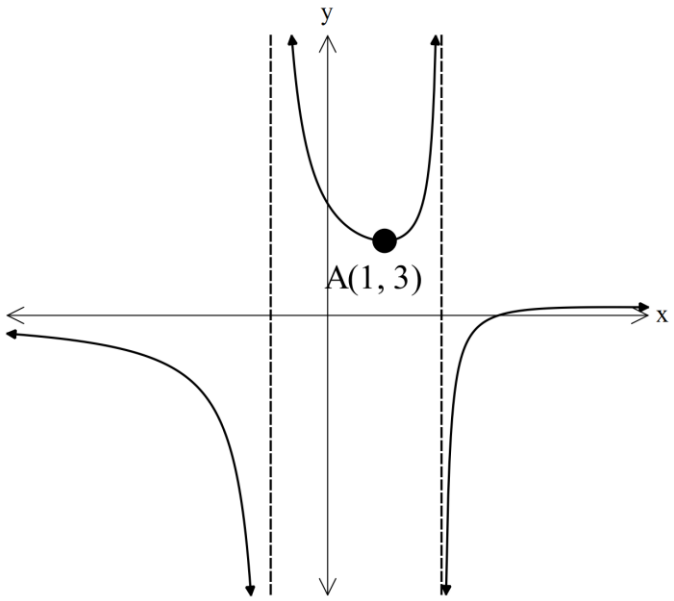
ii. x = 3

iii. x = 4

iv. x = 5

QUESTION 2 [3, 3 marks]

Consider the graph of $f(x) = \dfrac{3x - 9}{x^2 - x - 2}$ shown below with a local minimum at A(1, 3)



a) Show that $f'(x) = \dfrac{-3(x-1)(x-5)}{(x^2 - x - 2)^2}$

b) Hence, or otherwise, determine the coordinates of the local maximum value of f(x).

QUESTION 3 [3 marks]

The volume of a solid sphere is given by $\frac{4}{3}\pi r^3$ where r is the radius. If the radius is increased from 2 cm to 2.1 cm, use the incremental formula to find the approximate increase in volume. Give your answer simplified in terms of π.

QUESTION 4 [1, 2, 3, 2 marks]

The motion of a body is determined by $x = t^3 - 3t^2 - 9t + 4$, where x is measured in cm and t is in seconds. Find

a) The velocity-time equation

b) The acceleration-time equation

c) The time when the body is at rest

d) The acceleration when the body is at rest

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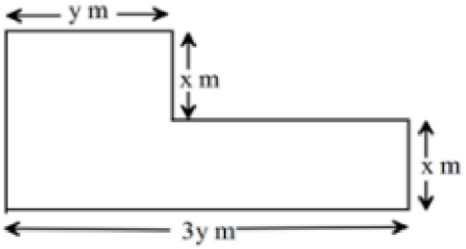
Section 2: Resource Rich 25 marks 25 minutes

QUESTION 5 [4 marks]

The graph of the function with rule $y = \frac{k}{2(x^3 + 1)}$ has gradient 1 when $x = 1$. Find the value of k.

QUESTION 6 [1, 1, 1, 2 marks]

A flower bed is to be L-shaped, as shown in the diagram. Its perimeter is 48 m



a) Write down an expression for the area, A m^2, in terms of y and x

b) Find y in terms of x

c) Write down an expression for A in terms of x.

d) Find the values of x and y that give the maximum area

QUESTION 7 [3 marks]

A coat of paint of thickness 0.05 cm is to be applied uniformly to the faces of a cube of edge 30 cm. Use calculus methods to find the amount of paint required for the job.

~~QUESTION 8 [4 marks]~~

~~The length of time, in seconds, a certain individual takes to learn a list of n items is approximated by $f(x) = 4n\sqrt{n-4}$. Use calculus to find the percentage increase in time taken when the number of items in the list is increased by 1%~~

QUESTION 9 [2, 1, 4 marks]

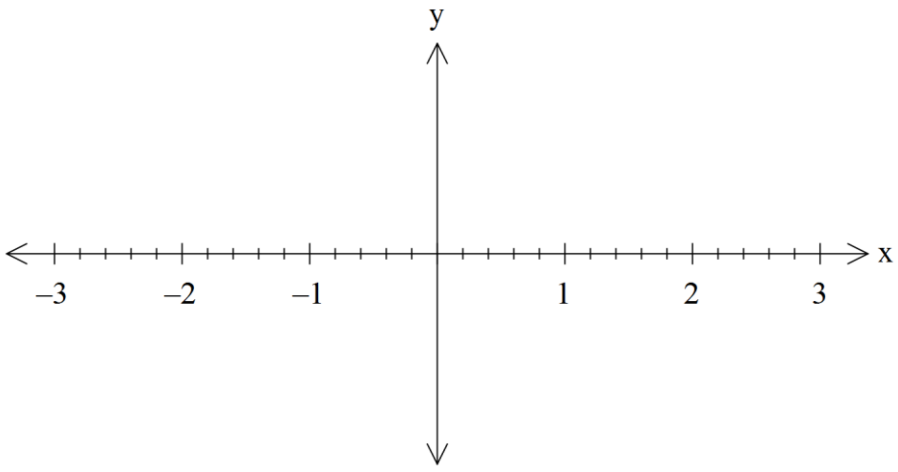
A POLYNOMIAL FUNCTION $f(x) = ax^4 + bx^2 + c$, where a, b and c are real constants, has the following features:

- f(x) = 0 only for x = -2 and x = 2
- f'(x) = 0 only for x = -1, x = 0 and x = 1
- f'(x) > 0 only for -1 < x < 0 and x > 1
- f''(0) < 0

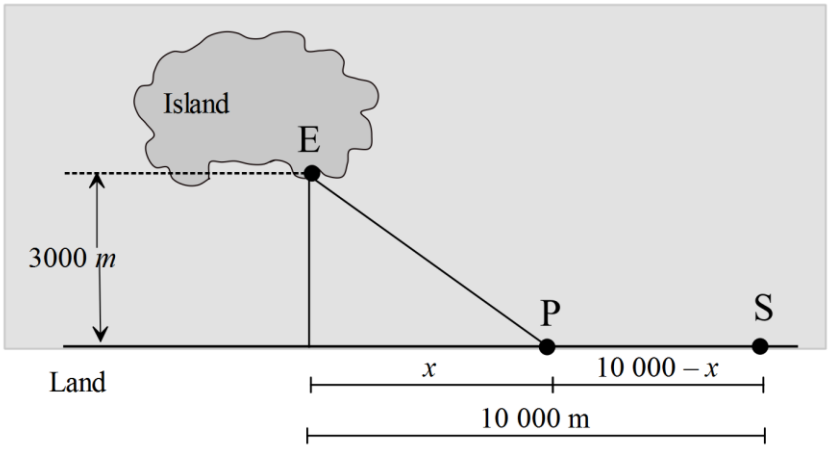
a) At the point where the curve intersects the y-axis, is it concave up or concave down? Explain your answer

b) Is c positive or negative? Explain your answer

c) Sketch a possible graph of the function on the axes below



Question 10 [1, 2, 3 marks]



In the accompanying diagram, S represents the position of a power relay station located on a straight coast and E shows the location of a marine biology experimental station on an island. A cable is to be laid connecting the relay station with the experimental station. The cost of running cable on land is \$1.50 per metre and the cost of running the cable under water is \$2.50 per metre. Locate the point P that will result in a minimum cost.

a) State the distance from E to P in terms of x

b) State the cost of the cabling in terms of x

c) Find the value of x that will minimise the cost